PART 1: MULTIPLE-CHOICE PROBLEMS

Each problem is worth 4 points: NO partial credit will be given. Calculators may NOT be used on this part. ScanTron forms will be collected after 1 hour.

1. $\tan(\cos^{-1}(\sqrt{3}/2)) =$

   (a) 2

   (b) $\frac{1}{2}$

   (c) $\sqrt{3}$

   (d) $\frac{1}{\sqrt{3}}$

   (e) $\frac{2}{\sqrt{3}}$

2. $\lim_{x\to 0} \frac{\ln(1+x)-x}{x^2} =$

   (a) 2

   (b) -2

   (c) $\frac{1}{2}$

   (d) $-\frac{1}{2}$

   (e) 1

3. $\frac{d}{dx}\sin^{-1}(2/x) =$

   (a) $-\frac{2}{\sqrt{x^4-4x^2}}$

   (b) $\frac{2}{\sqrt{x^4-4x^2}}$

   (c) $-\frac{2}{\sqrt{x^4+4x^2}}$

   (d) $\frac{2}{x^2+4}$

   (e) $\frac{x}{\sqrt{x^2-4}}$

4. If $f''(x) = 4x + 2$, $f'(0) = 1$ and $f(0) = -3$, then $f(3) =$

(a) 0

(b) 9

(c) 16

(d) 24

(e) 27

5. The table gives the values of a function obtained from an experiment:

x	0.0	0.5	1.0	1.5	2.0
$f(x)$	1.0	0.5	0.2	0.1	0.0

Find the Riemann sum for $\int_0^2 f(x)\,dx$ using four equal subintervals with right endpoints.

(a) 0.40

(b) 0.90

(c) 1.00

(d) 1.05

(e) 1.10

6. Evaluate $\lim_{x\to 0} (1+3x)^{\cot(x)}$

(a) $\ln 2$

(b) e^2

(c) e^3

(d) e^4

(e) 1

7. Which statement describes $f(x) = x^3 - 3x^2$?

(a) f has a critical value at $x = 3$.

(b) f is increasing on $(0, 2)$.

(c) f has a relative minimum at $x = 0$.

(d) f has a relative minimum at $x = 2$.

(e) f is concave upward on $(0, 1)$.

8. Find the minimum value of $x + 2y$ given that $xy = 50$ and x and y are positive.

(a) 10

(b) 15

(c) 20

(d) 25

(e) 30

9. If $\int_0^1 f(x)\,dx = 7$ and $\int_0^1 g(x)\,dx = 2$, then $\int_0^1 [f(x) - 4g(x)]\,dx =$

(a) 0

(b) 1

(c) -1

(d) 2

(e) -2

10. If $F(x) = \int_0^{\sin x} \frac{1}{1+t^4}\, dt$, then $F'(\pi/3) =$

(a) $\frac{16}{25}$

(b) 1

(c) $\frac{8\sqrt{3}}{25}$

(d) $\frac{8}{25}$

(e) 2

11. If $y = x^{\cos x}$, then $y' =$

(a) $x^{\cos x}\left[(\ln x)(\sin x) + \frac{\cos x}{x}\right]$

(b) $x^{\cos x}\left[-(\ln x)(\sin x) + \frac{\cos x}{x}\right]$

(c) $x^{-\sin x}$

(d) $-\sin x(\cos x)x^{\cos x - 1}$

(e) $x^{\cos x}\left[-(\ln x)(\sin x) + \frac{\sin x}{x}\right]$

PART 2: WORK-OUT PROBLEMS

Each problem is worth 8 points. Detailed analytic solutions must be provided. Partial credit is possible. Calculators are permitted ONLY AFTER the ScanTrons are collected.

12. A colony of bacteria grows at a rate proportional to its size. The initial population is 10^8. At the end of 10 minutes the population has increased to 1.03×10^8.

    (a) Find the value of the proportionality constant k. (4 points)

    (b) How long in minutes does it take the population to double its size? (4 points)

13. Find the absolute (global) maximum value of $g(x) = x^3 - 3x^2 - 9x$ on $[-2, 4]$.

14. Find the derivative of $f(x) = e^{-2x} \tan^{-1}(\ln x)$

15. Let $g(x) = \frac{1}{4}x^4 - \frac{1}{6}x^6$.

(a) Find the regions where $g(x)$ is increasing or decreasing. *Clearly justify your answer!* (4 points)

(b) Find the regions where $g(x)$ is concave up or concave down. *Clearly justify your answer!* (4 points)

16. Evaluate the following integrals. *Clearly show your reasoning!*

(a) $\displaystyle\int_{\ln 2}^{\ln 9} e^{x}\, dx =$ (4 points)

(b) $\displaystyle\int_{2}^{5} \frac{x+1}{x}\, dx =$ (4 points)

17. A box with an open top is to be constructed from a square piece of cardboard, 6 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

18. A car braked with a constant deceleration of 36 ft/sec^2, producing skid marks measuring 112.5 ft before coming to a stop. How fast in feet per second was the car traveling when the brakes were first applied?